Exponential Growth and Decay Problems

Equation for Exponential Models -- $f(x) = a(b)^x$

- a **is the initial amount or population**
- b **is the growth rate or decay rate**
- x **Is the amount of time**

Examples

1. The number of a certain bacteria doubles every hour. The initial population in a culture of this bacteria is 36.
   a) Determine the exponential model for the number of bacteria after t hours.
   b) Determine the number of bacteria after 8h.

a) The **growth rate** is 2 because the population *doubles* each hour. The **initial population** is 36.
The exponential model is given by: $P = 36(2)^t$

b) $P = 36(2)^8 = 9216$. The population after 8 hours is 9216 bacteria.

2. The population of a small town appears to be increasing exponentially. Town planners need a model for predicting the future population. In 1980, the population was 35000, and in 1990, the population was 57010.
   a) Create an algebraic model for the town's population growth.
   b) Check your model by using the fact that the population in 1995 was 72800.
   c) What will the population be in 2010?

a) The initial population was **a = 35000**
1980 is Year 0, 1990 is Year 10
In Year 10, the population was 57010
A model for population growth is found by calculating the growth factor.

$$57010 = 35000b^{10}$$
$$1.628 = b^{10}$$
$$\sqrt[10]{1.628} = b$$
$$b = 1.0499$$

Then $P = 35000(1.0499)^t$

b) The population in 1995 (Year 15)

$$P = 35000(1.0499)^t$$
$$= 35000(1.0499)^{15}$$
$$= 72658$$

The model is a good approximation of the actual population in 1995. Errors could be due to changes in population growth patterns or rounding.

c) The population in 2010 (Year 3):

$$P = 35000(1.0499)^t$$
$$= 35000(1.0499)^{30}$$
$$= 150836$$

3. An isotope of radium is used by a hospital for cancer radiation. The half-life of this radium is 1620 years. If the hospital initially had 10 mg, how much will they have after 50 years?

Half-Life is the amount of time it takes for a population or substance to reduce by half (or reduce to 50% of its previous amount). The exponential model for half-life is $A = a_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ where a_0 is the initial amount, h is the time to reduce by half.

The original amount is a = 10 mg.
The half-life is 1620 years
t is the number of years
A is the amount (in mg)

A model for exponential decay is $A = 10\left(\frac{1}{2}\right)^{\frac{t}{1620}}$

After 50 years, the amount remaining will be $A = 10\left(\frac{1}{2}\right)^{\frac{50}{1620}} = 9.78\,\text{mg}.$

Additional Problems:

1. The population of a city is 810000. If it is increasing by 4% per year, estimate the population in four years.

2. A painting, purchased for $10000 in 1990, increased in value by 8% per year. Find the value of the painting in the year 2000.

3. A river is stocked with 5000 salmon. The population of salmon increases by 7% per year.
   a. Write an expression for the population t years after the salmon were put into the river.
   b. What will the population be in 3 years? 15 years?
   c. How many years does it take for the salmon population to double?

4. A house was bought six years ago for $175000. If real-estate values have been increasing at a rate of 4% per year, what is the value of the house now?

5. A used-car dealer sells a five-year old car for $4200. What was the original value of the car if the depreciation is 15% per year?

6. In the early 1990s, the Canadian dollar was declining in value due to inflation at the rate of 8.3% per year. If the situation continued, what would the dollar be worth five years later?

7. To determine whether a pancreas is functioning normally, a tracer dye is injected. A normally functioning pancreas secretes 4% of the dye each minute. A doctor injects 0.50g of the dye. Twenty minutes later, 0.35g remains. If the pancreas were functioning normally, how much dye should remain?

8. If a bacteria population doubles in 5 days,
   a. When will it be 16 times as large?
   b. When was it ½ of its present population?
   c. When was it ¼ of its present population?
   d. When was it 1/32 of its present population?

9. Inflation is causing things to cost roughly 2% more per year.
   a. A bag of milk costs $3.75 now. Estimate its cost in five years.
   b. A movie ticket costs $8.50 now. If inflation continues at 2% per year, when will the ticket costs $10.0? How long ago did the movie ticket cost $4.25?

10. An element is decaying at a rate of 12%/h. Initially we have 100g.
    a. How much remains after 10h?
    b. How much remains after 30h?
    c. When will there be 40g left?

11. A population of bacteria, initially 1000, is growing. The size of the population is measured every hour. The results are shown in the table below.

Number of Time Intervals	Bacteria Population
0	1000
1	1135
2	1307
3	1490
4	1696
5	1957
6	2228

a. Draw a scatter plot of the data. Include labels and scales on the axes.
b. Determine the equation for the curve of best fit (accurate to 3 decimal places).
c. Use the equation to estimate the bacteria population in 10 hours.
d. Predict when there would be 10000 bacteria. (Hint: How many time intervals will this take?)

12. The table below shows the carbon dioxide concentration in the atmosphere in parts per million. We will count in intervals of 20 years from 1860.

Year	Time Interval	Carbon-Dioxide Concentration (in parts per million)
1860	0	294
1880	1	296
1900	2	300
1920	3	307
1940	4	308
1960	5	319
1980	6	340
2000	7	377

a. Draw a scatter plot of the data. Find the exponential equation for the curve of best fit (accurate to 3 decimal places)
b. Using your equation, estimate the carbon dioxide concentration in 1930 and in 1990.
c. If the trend continues, predict when the concentration will be 390 parts per million.