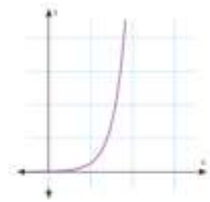
Key Concepts of Exponential Functions

3.1 The Nature of Exponential Growth

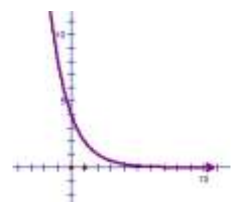
Properties of Exponential Growth



- As the independent variable increases by a constant amount, the dependent variable increases by a common factor.
- The graph increases at an increasing rate.
- The finite differences exhibit a repeating pattern: the ratio of the successive finite differences is constant.
- The equation of an exponential decay function is $y = a(b)^x$ where $b > 1$.

Any non-zero real number raised to the exponent zero is equal to 1: $b^0 = 1$ for $b \in R, b \neq 0$.

3.2 Exponential Decay: Connecting to Negative Exponents

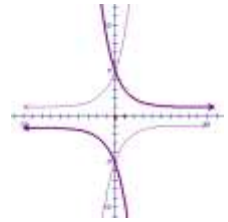


- Exponential decay functions have the following properties:
  - As the independent variable increases by constant amount, the dependent variable decreases by a common factor.
  - The graph decreases at a decreasing rate.
  - They have a repeating exponential pattern of finite differences: the ratio of successive finite differences is constant.
  - The equation of an exponential decay function is $y = a(b)^x$ where $0 < b < 1$.
- A power involving a negative exponent can be expressed using a positive exponent: $b^{-n} = \frac{1}{b^n}$ for $b \in R, b \neq 0$.
- The exponent rules hold for powers involving negative exponents.
- Rational expressions raised to a negative exponent can be simplified: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$ for $a, b \in R, a, b \neq 0$.

3.3 Rational Exponents

- A power involving a rational exponent with numerator 1 and denominator *n* can be interpreted as the nth root of the base: For $b \in R, b^{\frac{1}{n}} = \sqrt[n]{b}$. If n is even, b must be greater than or equal to 0.
- You can evaluate a power involving a rational exponent with numerator m and denominator m: For $b \in R, b^{\frac{m}{n}} = \sqrt[n]{b^m}$. If n is even, b must be greater than or equal to 0.
- The exponent rules hold for powers involving rational exponents.

3.4 Properties of Exponential Functions



- The graph of an exponential function of the form $y = ab^x$ is
  - Increasing if $a > 0$ and $b > 1$
  - Decreasing if $a > 0$ and $0 < b < 1$
  - Decreasing if $a < 0$ and $b > 1$
  - Increasing if $a < 0$ and $0 < b < 1$

- The graph of an exponential function of the form $y = ab^x$, where $a > 0$ and $b > 0$, has
  - Domain $x \in R$
  - Range $y \in R, y > 0$
  - A horizontal asymptote at $y = 0$
  - A y-intercept at a
- The graph of an exponential function of the form $y = ab^x$, where $a < 0$ and $b > 0$, has
  - Domain $x \in R$
  - Range $y \in R, y < 0$
  - A horizontal asymptote at $y = 0$
  - A y-intercept at a
- You can write an equation to model an exponential function if you are given enough information about its graph or properties
- Sometimes is makes sense to restrict the domain of an exponential model based on the situation it represents.

3.5 Transformations of Exponential Functions

- Exponential functions can be transformed in the same way as other functions.
- The graph of $y = ab^{k(x-d)} + c$ can be found by performing the following transformations on the base $y = b^x$:
  - Horizontal and Vertical Translations
  - Vertical Stretches, Translations, and Reflections
  - Horizontal Stretches, Compressions, and Reflections
- Some exponential functions can easily be written using different bases. For example, $y = 2^{4x}$ is equivalent to $y = 16^x$.