MCR 3U – Unit 3: Exponential Functions
Blackline Master Answer Key

3.1 Nature of Exponential Growth #1,5,7,8,10

1a)

Day	Population	First Differences	Second Difference
0	25		
		50	
1	75		100
		150	
2	225		300
		450	
3	675		900
		1350	
4	2025		

1b) Yes because the population triples

1c) They have the same common ratio as the population

1d) 3rd and 4th differences will have the same common ratio (although more data (days) is needed)

5a) B

5b) 15 is the initial population, 4 is the growth factor (quadruples), n is the number of days

7a) Exponential
7b) Linear
7c) None
7d) Quadratic

8a) Exponential
8b) Linear
8c) Quadratic
8d) Exponential
8e) Quadratic

10a) $p = 32(2)^t$ 10c) 128 10d) 4096

3.2 Exponential Decay: Connecting to Negative Exponents #1-5,7,8,10

1a) $\frac{1}{7}$
1b) $\frac{1}{10^2}$
1c) $\frac{1}{a^4}$
1d) $\frac{1}{mn}$
1e) $\frac{-1}{4}$
1f) $\frac{1}{2b}$

2a) a^{-3}
2b) $2x^{-5}$
2c) $-x^{-9}$
2d) $\frac{2}{5}b^{-6}$

3a) $\frac{1}{25}$
3b) $\frac{1}{81}$
3c) $\frac{1}{1000}$
3d) $\frac{1}{16}$
3e) $-\frac{1}{16}$
3f) $-\frac{1}{64}$
3g) $\frac{4}{81}$
3h) $-\frac{11}{48}$

4a) 25
4b) $\frac{1}{512}$
4c) 729
4d) $\frac{1}{36}$
4e) 7
4f) 64

5a) a^4
5b) $-\frac{12}{v^7}$
5c) a^{10}
5d) $\frac{3}{m^2}$

7a) 6
7b) $\frac{49}{16}$
7c) $\frac{64}{27}$
7d) $\frac{8n^2}{125m^6}$

8a)

Days # of 20 Day Intervals	Sample (mg)
0	40
1	20
2	10
3	5
4	2.5

8b) $f(x) = 40\left(\frac{1}{2}\right)^t$
8d) 3.54 mg
8e) 72.87 days
10. $2720.63

3.4 Properties of Exponential Functions #1,2,5,7,8,10

1a) B
1b) D
1c) C
1d) A

2a) C
2b) A
2c) D
2d) B

5. $y = 6(2)^x$

7a) C
7b) 9.5 mg

8a) i) $x \in \Re$ ii) y > 0 iii) (0,1) iv) decreasing for all x v) x-axis
8b) i) $x \in \Re$ ii) y > 0 iii) (0,3) iv) increasing for all x v) x-axis
8c) i) $x \in \Re$ ii) y < 0 iii) (0,-1) iv) increasing for all x v) x-axis

10b) i) asymptote x-axis, no x-intercept, decreasing, similar shape
ii) domain, asymptotes, # of sections

c) Both graphs have y=0 as horizontal asymptote

3.5 Transformations of Exponential Functions

#1-4,6,9,11

1.

The Roles of the Parameters a, k, d, and c in Exponential Functions of the Form $y = ab^{k(x-d)} + c$ $(b > 0, b \neq 1)$	
Role of c	**Transformation on Graph of $y = b^x$**
$c > 0$	Translation c units up
$c < 0$	Translation c units down
Role of d	
$d > 0$	Translation d units right
$d < 0$	Translation d units left
Role of a	
$a > 1$	Vertical stretch by a factor of a
$0 < a < 1$	Vertical compression by a factor of a
$-1 < a < 0$	Vertical compression by a factor of $\lvert a \rvert$ and a reflection in the x-axis
$a < -1$	Vertical stretch by a factor of $\lvert a \rvert$ and reflection in the x-axis
Role of k	
$k > 1$	Horizontal compression by a factor of $\frac{1}{k}$
$0 < k < 1$	Horizontal stretch by a factor of $\frac{1}{k}$
$-1 < k < 0$	Horizontal stretch by a factor of $\frac{1}{\lvert k \rvert}$ and a reflection in the y-axis
$k < -1$	Horizontal compression by a factor of $\frac{1}{\lvert k \rvert}$ and a reflection in the y-axis
Domain and Range of $y = ab^{k(x-d)} + c$	
The domain is always $\{x \in \mathbb{R}\}$.	i) When the graph is below its horizontal asymptote the range is $\{y \in \mathbb{R}, y < c\}$. ii) When the graph is above its horizontal asymptote the range is $\{y \in \mathbb{R}, y > c\}$.

2a) Up 3

2b) Right 2

2c) Left 1

2d) Right 4, Down 6

4a) Vertical Compress

4b) Horiz Compress

4c) Vertical Reflection in x-axis

4d) Horiz Reflection and Stretch

6a) $y = 11^{-x}$

6b) $y = 4(11)^x$

6c) $y = 11^{\frac{1}{3}x}$

6d) $y = -11^{5x}$

9a) Vertical Reflection, Horizontal Compress, Down 7 $y = -3^{4x} - 7$

b) $x \in \Re, y < -7, y = -7$

11b) y = 18

c) approx 33min

3.6 Applying Exponential Models

#1,3,4,5,11

1 A $y = 2 \times 2.2^x$

B $y = 6 \times 1.6^x$

C $y = 9 \times 0.8^x$

3a) Yes – increasing slowly but gets faster

3c) $V(n) = 150 \times 1.05^x$

3d) $584.40

3e) approx 17 years

4a) $C = 100(.98)^t$

4b) i) 60% ii) 36% iii) 22% iv) 13%

4c) Half-life is 34 days

5c) 7.4

5d) $78917.6

5e) 2004

Chapter 3 Review

#1-19

1a) C

b) 85 is initial population, 3^n is the constant ratio for tripling

2a) 1

b) 1

c) 1

d) -1

3a) i) exponential ii) linear iii) quadratic

3b) i) constant ratio of 1st and 2nd differences equal

ii) first differences equal

iii) second differences equal

9a) $(-3125)^{\frac{4}{5}}$

b) $32^{\frac{3}{5}}$

10a) $\sqrt[5]{(-32)^4}$, 16

b) $\sqrt[3]{\left(\frac{1}{343}\right)^2}, \frac{1}{49}$

c) $\sqrt[3]{(-125)^2}$, 25

11a) $16^{-\frac{1}{2}} = \frac{1}{4}$

b) $81^{\frac{3}{2}} = 729$

c) $256^{\frac{1}{8}} = 2$

12a) $s^{\frac{5}{12}}$

b) $m^{\frac{2}{7}} n^{\frac{8}{15}}$

c) $\frac{1}{k^{\frac{3}{14}}}$

d) $\frac{3}{32} v^{\frac{9}{10}}$

13a) $s = \frac{2\sqrt{A}}{3^{\frac{1}{4}}}$

b) 5.39 m

14a) i) $x \in \Re$ ii) y > 0 iii) (0,1) iv) decreasing v) y = 0

4a) $\frac{1}{x^3}$ b) $\frac{3}{b^2}$

5a) w^{-4} b) $-36b^{-8}$

6a) $\frac{1}{216}$ b) $\frac{6}{125}$ c) $\frac{1}{12}$

d) 8 e) $\frac{125}{216}$ f) $\frac{49}{64}$

7a) $\frac{b^{10}}{9}$ b) $-\frac{b^6}{8a^6}$ c) $\frac{x^{18}}{729}$ d) $\frac{243d^{10}}{32c^{20}}$

8a) $\frac{16}{9}$ b) $\frac{2187}{128}$ c) -5 d) 9

14b) i) $x \in \Re$ ii) y > 0 iii) (0,4) iv) increasing v) y = 0

14c) i) $x \in \Re$ ii) y < 0 iii) (0,-1) iv) increasing v) y = 0

15a) $A = 28\left(\frac{1}{2}\right)^{\frac{t}{5}}$ b) Amount reduces by ½ every 5 days

15c) Decreasing; (0,28) is the y-intercept; horiz asymptote at y = 0; 15d) 4 mg

Domain: $t \in \Re, t > 0$, Range: $A \in \Re, A > 0$

17a) Vertical Stretch by2, Horiz Stretch by 3, Shifted Right 5; $f(x) = 2(0.5^{\frac{1}{3}(x-5)})$

17b) $x \in \Re; y \in \Re, y > 0$, Horiz Asymp y = 0

18a) $f(x) = -\frac{1}{3}\left(\left(\frac{1}{4}\right)^{2(x+4)}\right) + 6$ b) $x \in \Re; y \in \Re, y < 6$ Horiz Asymp y = 6

19 b) $P(t) = 24(1.014)^t$ b) 31.3 million c) 36.9 million d) 2018