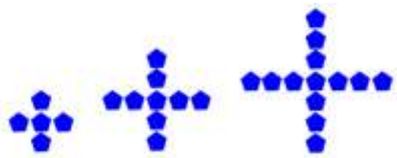
Sequence & Series

Discrete Functions - ARITHMETIC SEQUENCES

Determine the patterns in the following. Can you also determine what comes next?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



1 1 2 3 5 8 13 …

4, 8, 12, 16, 20, …

20, 10, 5, 2.5, …

-3 + 5 + 13 + …

8 – 16 + 32 – 64 +…

Sequence: an ordered list of numbers or terms; e.g., 1, 2, 4, 8, 16, 32, 64, 128, 256.

Term: each number is a term, and individual terms in a sequence are represented by t_1 for the first term, t_4 for the fourth term, or, in general, by t_n for the n^{th} term.

General Term: If there is a pattern to the sequence, then the general term for a sequence is represented by t_n , or a method of generating the terms in the sequence; e.g., $t_n = 2^{n-1}$

⇨ Describe the pattern in each sequence. Explain any differences you see in each sequence.

5, 7, 9, 11, …

Increase by adding 2

2, 6, 18, 54, …

Increase by multiple of 3

Arithmetic Sequence:

The first term, t_1, is denoted by the letter **a**
The difference that occurs in the pattern of the sequence is denoted by **d**

Each term after the first is found by adding a constant, d, to the preceding term.

Sequence & Series

Finding a formula for terms in an arithmetic sequence

1. Identify a and d for the sequence 5,7,9,11,… $a = 5, d = 2$

2. How would you find the each term in the arithmetic sequence? (You know the terms because this is an easy example…..but if you needed to do a calculation, what would it be?)

t_1	$= a$			$= 5$
$t_2 = t_1 + d$	$= a + d$		$= 5 + 2$	$= 7$
$t_3 = t_2 + d$	$= a + d + d$	$= a + 2d$	$= 5 + 2(2)$	$= 9$
$t_4 = t_3 + d$	$= a+d+d+d$	$= a + 3d$	$= 5 + 3(2)$	$= 11$
$t_5 = t_4 + d$	$= a+d+d+d+d$	$= a + 4d$	$= 5 + 4(2)$	$= 13$

3. What is the general formula, t_n for this sequence? (hint -- look at the 4th column)
   $t_n = 5 + (n-1)2$

For an Arithmetic Sequence, the general term t_n is $t_n = a + (n-1)d$

EXAMPLES

1. Find the 6th, 12th and nth term of an arithmetic sequence where a = 11, d = -5.
   $t_n = 11 + (n-1)(-5)$ $t_6 = 11 + (5)(-5) = -14$
   $t_{12} = 11 + (11) = -44$

2. Find a formula for the sequence 3,9,15,21, …

   $t_n = 3 + (n-1)(6)$ or $t_n = 6n - 3$

3. How many terms are in the sequence -7, -3, 1,….. 29?
   $t_n = -7 + (n-1)(4)$ where the last term $t_n = 29$
   $29= -7 + (n-1)(4)$
   $36=(n-1)(4)$ Solve for n
   $9 = n - 1$
   $10 = n$ Therefore, there are 10 terms

4. Given that t_5=11 and t_{12}= 25 in an arithmetic sequence, find a formula for t_n.

   $t_5 = 11 = a + 4d$ Arithmetic sequence formula for the given terms
   $t_{12} = \underline{25 = a + 11d}$
   $14 = 7d$ Subtract equations (elimination)
   $2 = d$
   $11 = a + 4(2)$ Sub d = 2 into one of the given equations
   $a = 3$

   $t_n = 3 + (n-1)2$ or $t_n = 1 + 2n$

p. 441 #3,4ace, 5acegi, 6acegi, 7ace, 8ace, 12, 15, 22

Arithmetic Sequences Practice

Sequence & Series

1. Find the next two numbers in each pattern.

   A) 70, 66, 62, 58 ,54, 50

   B) 3, 9, 15, 21, 27, 33

   C) 9, 16, 23 , 30, 37

   D) 3, 2.5, 2, 1.5, 1, 0.5

2. Find an expression for the general term for the sequences listed above.

   A) t_n = 70 + (n-1)(-4)

   B) t_n = 3 + (n-1)(6)

   C) t_n = 9 + (n-1)(7)

   D) t_n = 3 + (n-1)(-0.5)

3. Determine t_3 , t_7 , and t_n for the arithmetic sequences with properties listed below

   A) a = 2 and d = 5

   B) t_{14}= 46, and t_{19}= 100

A) $t_n = 2 + (n-1)(5)$
$t_3 = 2 + (3-1)(5) = 12$
$t_7 = 2 + (7-1)(5) = 32$

B) $t_{14} = a + 13d = 46$
$t_{19} = a + 18d = 100$
$5d = 54$
d = 10.8
$a + 13(10.8) = 46$
$a = 46 – 140.4$
a = - 94.4

$t_n = -94.4 + (n-1)(10.8)$
$t_3 = -94.4 + (3-1)(10.8) = -72.8$
$t_7 = -94.4 + (7-1)(10.8) = - 29.6$

4. Determine the number of terms in the arithmetic sequence -45, -42, -39, . . . , 15.

$a = -45, d = 3, t_n = 15$
$t_n = -45 + (n-1)(3)$
$15 = -45 + (n-1)(3)$
$60 = (n-1)(3)$
$20 = n-1$
$n = 21$

Sequence & Series

Discrete Functions - GEOMETRIC SEQUENCES

Compare the following sequences

4, 6, 8, 10,... +2
4, 8, 16, 32, ... x 2
4, 2, 1, ½, ... ÷2

Define GEOMETRIC SEQUENCE:

The next term increases by a common multiple (or ratio)

Let a be the first term in the sequence.
Let r be the common ratio between the terms of a sequence.

Investigate and inquire p. 447

Read p. 447 and complete the following questions.

1.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
Frequency	27.5	55	110	220	**440**	**880**	**1760**	**3250**
Use Powers of 2	27.5	27.5×2	27.5×2^2	27.5×2^3	27.5×2^4	27.5×2^5	27.5×2^6	27.5×2^7
Use a and r	a	$a\,r^1$	ar^2	ar^3	ar^4	ar^5	ar^6	ar^7

2. What are the values of a and r for this sequence? → a = 27.5 , r = 2
3. What is the formula used to find t_5? $t_5 = 27.5 \times 2^4$
4. Formula for nth term in geometric sequence $t_n = 27.5 \times 2^{n-1}$
5. Write the formula for the nth term of the sequence -- 2, -10, 50, -250, …. $t_n = 2(-5)^{n-1}$

The terms of a geometric sequence are: **$a, ar, ar^2, ar^3, \ldots ar^{n-2}, ar^{n-1}$**

The formula for the nth term of a geometric sequence is: **$t_n = ar^{n-1}$**

Examples

1. Find the first 4 terms of the geometric sequence $t_n = -4(2)^{n-1}$ -4,8, -16, 32
2. Find a formula for the nth term in the sequence 8, 24, 72, …. $t_n = 8(3)^{n-1}$
3. Given 2 terms in a geometric sequence, $t_3 = 28$ and $t_4 = 56$, find t_n.
   $t_3 = ar^2 = 28$ r = 2 (divide the terms) $a(2)^2 = 28$ a = 7 $t_n = 7(2)^{n-1}$
   $t_4 = ar^3 = 56$
4. Given $t_2 = 4$, $t_4 = 64$, find t_9 and t_{10}
   $t_2 = ar^1 = 4$ → $r^2 = 16$ → r = +/-4 → a = +/-1 → $t_n = t_n = \pm1(\pm4)^{n-1}$
   $t_4 = ar^3 = 64$

Sequence & Series

Geometric Practice

1. Find the next two numbers in each pattern.

A) -16, -8, -4 ,-2,1

C) 16, -8, 4 , -2, 1

B) 3, 9, 27, 81,243

D) 3, 1.5, 0.75 , 0.375,0.1875

2. Find an expression for the general term for the sequences listed above.

A) $t_n = -16\left(\frac{1}{2}\right)^{n-1}$

C) $t_n = 16\left(-\frac{1}{2}\right)^{n-1}$

B) $t_n = 3(3)^{n-1}$

D) $t_n = 3\left(\frac{1}{2}\right)^{n-1}$

3. Determine t_3 , t_5 , and t_n for the geometric sequences with properties listed below

A) a = 5x and r = 2x

$t_n = 5x(2x)^{n-1}$

$t_3 = 5x(2x)^2 = 20x^3$

$t_5 = 5x(2x)^4 = 80x^5$

B) t_6= 6, and t_4= 24

$t_6 = ar^5 = 6$ Divide the terms $r^{-2} = 4$

$t_4 = ar^3 = 24$ $r = \pm\frac{1}{2}$

Solve for a $a\left(\frac{1}{2}\right)^3 = 24$ $a\left(-\frac{1}{2}\right)^3 = 24$

$a = 192$ $a = -192$

$t_n = \pm 192\left(\pm\frac{1}{2}\right)^{n-1}$

$t_3 = \pm 192\left(\pm\frac{1}{2}\right)^2 = \pm 48$

$t_5 = \pm 192\left(\pm\frac{1}{2}\right)^4 = \pm 12$

4. Determine the number of terms in the geometric sequence 4, 12, 36, . . . , 8748.

a= 4, r = 3, t_n = 8748

$8748 = 4(3)^{n-1}$

$2187 = 3^{n-1}$

$3^7 = 3^{n-1}$

$n = 8$

Additional Textbook Practice: p. 452 # 1-7 (ace), 9, 13

Sequence & Series

Arithmetic Series

1. Review investigate and inquire questions (pp. 465-6)
2. Read about a child genus – Karl Gauss (p. 466)
3. Give a definition and formula for Arithmetic Series.
   Sum of terms in an arithmetic sequence

4. How do you find the sum of a series if you are given the first few terms?

   Read Example 1 (p. 467) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

   Determine the sum of first 6 terms of each series

   a) 4 + 7 + 10 + …

   $$S_n = \frac{n}{2}\left[2(4) + (n-1)3\right]$$
   $$S_6 = \frac{6}{2}\left[2(4) + (6-1)3\right]$$
   $$= 3\left[8 + 5(3)\right]$$
   $$= 69$$

   b) a = −4, d = −3

   $$S_n = \frac{n}{2}\left[2(-4) + (n-1)(-3)\right]$$
   $$S_6 = \frac{6}{2}\left[2(-4) + (6-1)(-3)\right]$$
   $$= 3\left[-8 + 5(-3)\right]$$
   $$= -69$$

5. How do you find the sum of a series given the first few and the last term of the series? Read Example 2 (p. 467) $S_n = \frac{n}{2}\left[a + t_n\right]$

   Determine the sum of the first 10 terms

   a =2, t_{10} = 38

   $$S_{10} = \frac{10}{2}\left[2 + 38\right]$$
   $$= 5(40)$$
   $$= 200$$

6. Determine the sum of the arithmetic series 3 + 8 + 13 + …+ 58.
   (Hint: How many terms are in this series? First, find n.)

   $$t_n = 3 + (n-1)5$$
   $$58 = 3 + (n-1)5$$
   $$55 = (n-1)5$$
   $$11 = n - 1$$
   $$n = 12$$

   $$S_{12} = \frac{12}{2}\left[3 + 58\right]$$
   $$= 6(61)$$
   $$= 366$$

Suggested Text Problems: pp. 469 # 2– 5 (ace), 10, 23

Sequence & Series

Geometric Series

The sum of the terms of a geometric sequence is called a geometric series.

Andy's neighbours are planning on taking a vacation for two weeks (14 days). They have asked him to look after their cat and to water their plants. The neighbours have offered to pay him $5 per day or $0.01 the first day, $0.02 the second day, $0.04 the third day, $0.08 the fourth day, etc. Which method of payment should Andy choose?

Andy notices that the amounts he will earn each day if he chooses the second plan are the terms of the geometric sequence

$0.01, 0.01(2)^2, 0.01(2)^3, \ldots, 0.01(2)^{13}$

He uses the following method to calculate the amount that he will have at the end of two weeks.

$$S_{14} = 0.01 + 0.01(2) + 0.01(2)^2 + 0.01(2)^3 + \ldots + 0.01(2)^{13}$$
$$2S_{14} = 0.01(2) + 0.01(2)^2 + 0.01(2)^3 + \ldots + 0.01(2)^{13} + 0.01(2)^{14}$$
$$2S_{14} - S_{14} = 0.01(2)^{14} - 0.01$$
$$S_{14} = \frac{0.01[(2)^{14} - 1]}{2 - 1}$$
$$= \$163.83$$

Using this method, a formula can be developed to find the amount that Andy will earn if he works any number of days (n).

$$S_n = a + ar + ar^2 + ar^3 + \ldots.. + ar^{n-1}$$
$$rS_n = ar + ar^2 + ar^3 + \ldots.. + ar^{n-1} + ar^n$$
$$rS_n - S_n = ar^n - a$$
$$S_n(r-1) = a(r^n - 1)$$
$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

If the neighbours are away for 7 days which method of payment should Andy choose?

Sequence & Series

Geometric Series Practice

(See p. 474 – 475 Examples 1 – 3)

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$

Problem 1: Find S_{12} for 1+2+4+…

a = 1, r =2

$$S_{12} = \frac{1\left(2^{12}-1\right)}{2-1} = 4095$$

Problem 2: Find S_n if a = 625, r = 0.6, n = 5

a = 625, r =0.6

$$S_5 = \frac{625\left(0.6^5 - 1\right)}{0.6-1} = 1441$$

Problem 3: Find the sum of 1300+130+13+…..+0.0013
(Hint – First step: Use a geometric sequence to solve for the number of terms)

a = 1300, r =0.1

$$\begin{aligned} t_n &= 1300(0.1)^{n-1} \\ 0.0013 &= 1300(0.1)^{n-1} \\ 0.000001 &= (0.1)^{n-1} \\ 0.1^6 &= (0.1)^{n-1} \\ n &= 7 \end{aligned}$$

$$S_7 = \frac{1300\left(0.1^7 - 1\right)}{0.1-1} = 1444.4443$$

Textbook p. 476 #1-3 (ace), 10, 13