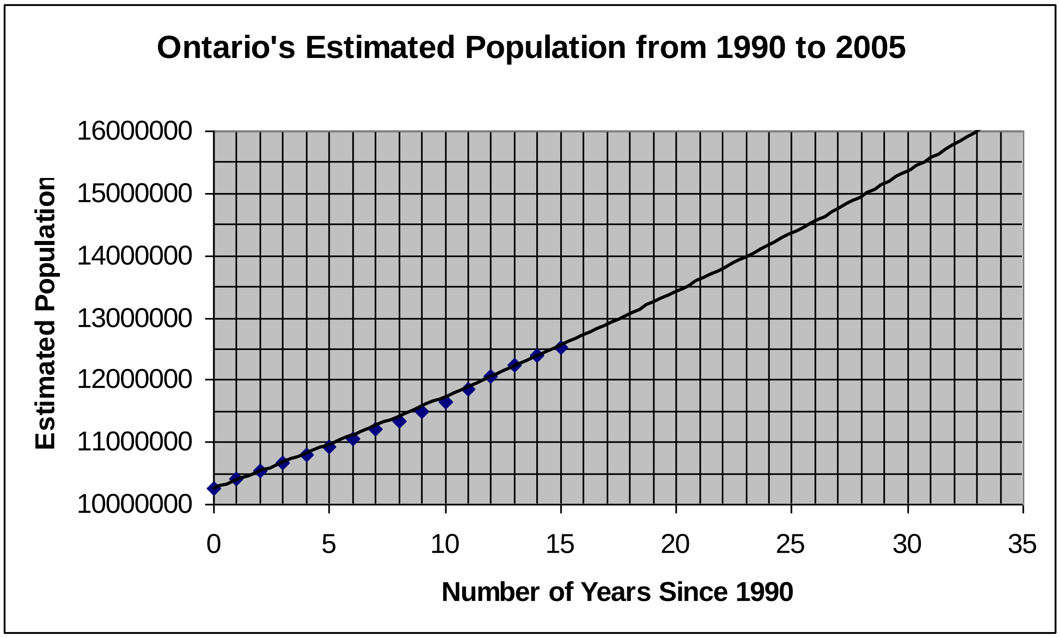
5.10.1 Using Graphical and Algebraic Models

Population Growth

The following graph shows Ontario's estimated population from 1990 to 2005 as found on the Statistics Canada website. The dots represent actual data. A curve of best fit has been added to the graph.



(Source: Statistics Canada)

1. Using the graph, estimate Ontario's population in 1995.

2. Using the graph, predict Ontario's population in 2010. Does this seem reasonable? What are you assuming about the growth pattern?

5.10.1 Using Graphical and Algebraic Models (continued)

3. Ontario's population is growing at a rate of approximately 1.35% per year. The estimated population, *y*, can be modelled by $y = 10269192(1.0135)^t$, where *t* is the number of years since 1990 and 10269192 was the estimated population in 1990.

   a. Use the equation modelling Ontario's population to calculate the estimated population in 1995. Compare this algebraic result to the graphical result from question 1.

   b. Use the equation modelling Ontario's population to calculate the projected population in 2010. Compare this algebraic result to the graphical result from question 2.

   c. When would you use a graphical model to convey mathematical information?

   d. When would you use an algebraic model (i.e., equation) to convey mathematical information?

4. Some people might think that the graph indicates this data is linear. Do you agree? Why or why not?

5. When will Ontario's population reach 15 million? Does this seem reasonable?

6. How would the graph and equation be different if the growth rate was1.5%?

5.10.1 Using Graphical and Algebraic Models (continued)

Rebound Height

When a basketball is correctly inflated, it rebounds to approximately 60% of the height from which it is dropped. A correctly inflated basketball is dropped from a height of 2.4 m and continues to bounce, each time rebounding to 60% of its previous height.

1. The rebound height of the basketball, *h*, can be modelled by the equation $h = 2.4(0.6)^n$, where *n* is the number of rebounds.

    a. Explain the meaning of 2.4 and 0.6 in this equation.

    b. Use the equation to determine the rebound height of the basketball after 5 rebounds.

2. Suppose the ball stops rebounding and begins to roll across the floor when it reaches a rebound height of 3 cm. How many times has the ball rebounded? Explain how you solved this problem.

3. What is the domain and range for the function modelling the rebound height of the basketball?

4. How would the equation change if:

    a. the ball was over-inflated and rebounded to 75% of its previous rebound height.

    b. the ball was dropped from an initial height of 2 m.

5.10.1 Using Graphical and Algebraic Models (continued)

5. Yvonne and Nancy are avid basketball players. After playing basketball outside on a hot summer day, they stop for a lemonade break. Yvonne sits down on the bench of the picnic table while Nancy stands on the bench on the opposite side. Nancy holds the basketball above her head and drops it onto the top of the picnic table from a height of 2.2 m above the top of the picnic table.

   a. Based on the information above, explain why the equation $h = 2.2(0.6)^n$ would model the rebound height of the basketball in relation to the top of the picnic table after *n* rebounds.

   b. The top of the picnic table is 70 cm above the patio. Explain how you think this would affect the rebound height if it is measured from the patio rather than from the top of the picnic table on which it is being bounced.

   c. The equation that models the height of the rebound in relation to the patio is $h = 2.2(0.6)^n + 0.7$.

      i. Calculate the rebound height, <u>in relation to the top of the picnic table</u> on the third bounce.

      ii. Calculate the rebound height, <u>in relation to the patio</u> on the third bounce.

      iii. How do the two values compare?

   d. Yvonne stands on the bench of a different picnic table. She holds the ball over her head and drops the ball onto the table from a height of 1.7 metres above the top of the table. If the top of the table is 80 cm above the patio, suggest how the equation modelling the rebound height in relation to the patio would change.

5.10.1 Using Graphical and Algebraic Models (continued)

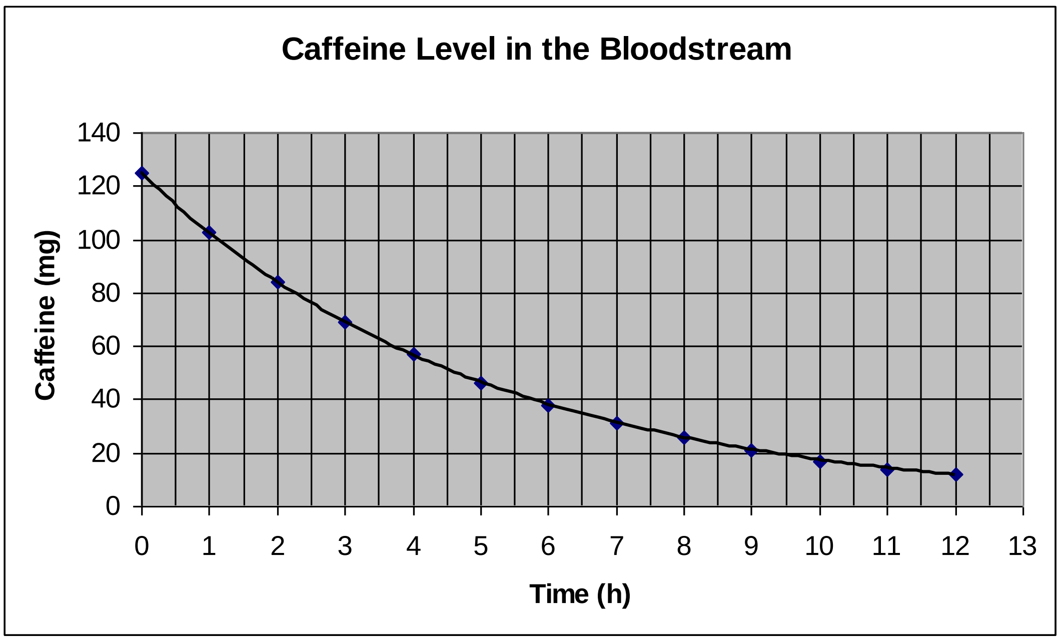
Caffeine Consumption

(Source: **Advanced Functions and Introductory Calculus**, Nelson, 2002, p. 128)

When you drink coffee, tea, or hot chocolate, or eat a chocolate bar, your body absorbs chemicals from these foods, including caffeine. The amount of caffeine in your bloodstream follows an exponential pattern over time.

The highest level of caffeine in the bloodstream occurs 15 min to 45 min after drinking a beverage or eating a food with caffeine. Then the level of caffeine begins to fall.

The following graph shows the caffeine level in the bloodstream of Peter, over time. Peter is an adult smoker who has consumed a cup of coffee (250 mL). The coffee contains 125 mg of caffeine that peaks in his bloodstream shortly after consumption. The graph starts at the time when the caffeine level peaks (i.e., $t = 0$ when the caffeine level peaks).



1. Using the graph, determine the amount of caffeine in Peter's bloodstream after 4.5 hours.

2. Using the graph, determine when Peter will have 20 mg of caffeine in his system.

3. The time it takes for half of the original amount of caffeine to remain in the bloodstream is called the half-life. Use the graph to determine the half-life of caffeine in Peter's bloodstream. Show your work on the graph.

The half-life from the graph is: ________________

The length of the half-life of caffeine is affected by a number of factors, including age. The following data shows the half-life of caffeine for a variety of factors.

Factor	Half-life
Adult non-smoker	5.5 h
Adult smoker	3.5 h
Woman who is six months pregnant	10 h to 18 h
Newborn baby	100 h
8-month-old baby	4 h
6-year old to 10-year-old child	2 h to 3 h

An appropriate model for the amount of caffeine in a person's bloodstream is

$$y = c\left(\tfrac{1}{2}\right)^{\frac{t}{h}}, \text{ or } y = c(0.5)^{\frac{t}{h}}$$

where
- *y* is the amount of caffeine in the bloodstream in mg,
- *c* is the initial amount of caffeine in mg,
- *t* is the number of hours since the caffeine level in the bloodstream has peaked
- *h* is the half-life of caffeine in hours (i.e., the amount of time for half of the caffeine to remain in the bloodstream)
- *0.5* indicates that the caffeine is decaying by a factor of ½ (hence 'half-life')

4. Jenny is an adult and does not smoke. She also consumes a cup of coffee (250 mL). Complete the following table by substituting values into the equation modelling caffeine level.

FIRST

Substitute the values for c and h into the equation $y = c(0.5)^{\frac{t}{h}}$ and write the resulting equation to calculate Jenny's caffeine level at various times (*t=0 to t=12*)

The equation is: ______________

t (hours)	*y* (in mg)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

5.10.1 Using Graphical and Algebraic Models (continued)

5. Graph Jenny's caffeine level over time on the same graph modelling Peter's caffeine level.

6. Compare Jenny and Peter's graphs. In your comparison, discuss the shapes of graphs, the type of functions, the y-intercepts, horizontal asymptotes, domain, and range.

7. Confirm that the half-life of caffeine in Jenny's bloodstream is 5.5 h by using the graph. Show your work on the graph.

8. A woman who is six months pregnant drinks a 250 mL cup of coffee. From the chart, the half-life is 10-18 hours. Assume a half-life of 14 hours. If she doesn't consume any more caffeine, would she have any caffeine left in her bloodstream 2 days later? If so, how much?