

MCR 3U – Unit 1 Performance Task

Laser Beams

A concert stage has a parabolic roof. The front edge of the roof is defined by the equation $h(x) = -\frac{1}{8}x^2 + 8$, where x is the horizontal distance from the centre and h is the height, both in metres. A vertical lighting tower is built at $x = 9$. Coloured laser lights are installed at various intervals going up the tower. The beams of light are to shine on the front edge of the roof, with their paths defined by the following equations.

Blue: $6x + 8y - 73 = 0$

Green: $x + 2y - 17 = 0$

Orange: $x + y - 10 = 0$

Red: $2x + 8y - 67 = 0$

- Sketch the graph of $h(x)$.
- Determine the coordinates of any point(s) of intersection of each laser beam with the edge of the roof.
- All but one of these laser beams share a common property. Describe the property.
- Determine the height of each light source on the tower.
- Determine an equation for the path of a fifth laser beam that is to be tangent to the edge of the roof at the vertex. Where should this light source be located on the tower?
- Consider the one laser light that does not share the common property. Keeping the location of the light source fixed, determine a new equation for the path of the laser light so that it now shares the common property. Where does it intersect the edge of the roof?