

NUMERICAL PROPERTIES OF POLYNOMIAL FUNCTIONS

What is the connection between finite differences and a polynomial function?

- Do 1.5.3 #1 - 5 (Question 4 should be $y = x^4$)

A polynomial of degree n has constant n^{th} differences

For example, $y = x^2 - 3 \rightarrow$

* Make sure you have enough data
(more needed as degree increases)

| X | Y | 1 st | 2 nd | 3 rd |
|----|----|-----------------|-----------------|-----------------|
| -2 | -2 | 4 | -6 | 6 |
| -1 | 2 | -2 | 0 | 6 |
| 0 | 0 | -2 | 6 | 6 |
| 1 | -2 | 4 | | |
| 2 | 2 | | | |

The differences have the same sign as the leading coefficient



n^{th} differences are equal to $a \times n!$

a is the leading coefficient. n is the degree of the polynomial
 $n!$ - means n factorial..... eg. $5! = 5 \times 4 \times 3 \times 2 \times 1$

Example

Find the constant differences of $y = -6x^3 + x$

$$3^{\text{rd}} \text{ diff} = \underset{\substack{\uparrow \\ \text{Leading coeff.}}}{-6} \times \underset{\substack{\uparrow \\ \text{degree}}}{3!} = -6 \times (3 \times 2 \times 1) = \underline{\underline{-36}}$$

Example

A polynomial has degree 5 and 5th difference of 1200.
Find the leading coefficient.

$$\begin{aligned} 1200 &= a \times 5! \\ &= a \times 120 \\ a &= \frac{1200}{120} \\ a &= 10 \end{aligned}$$