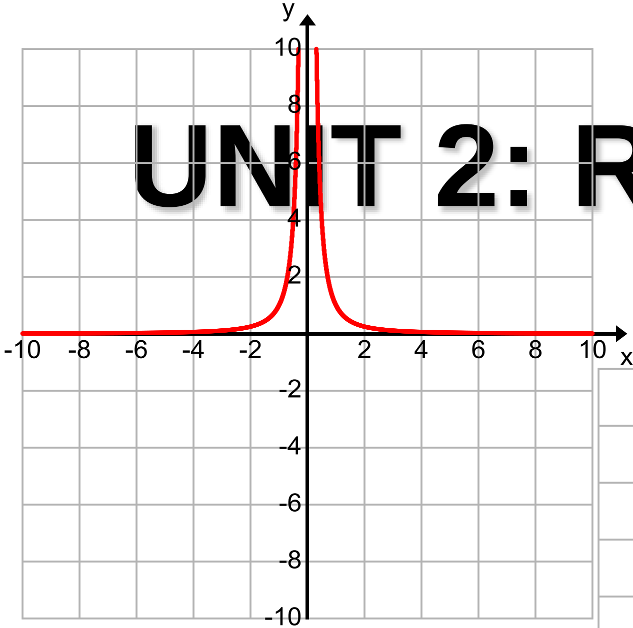
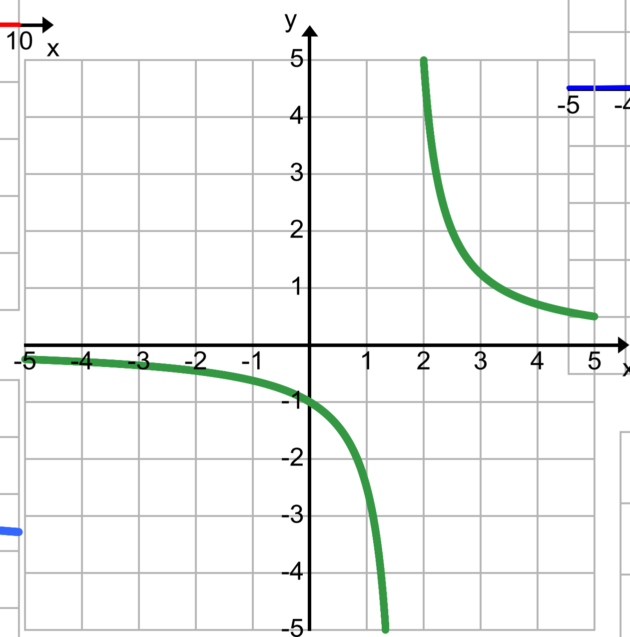
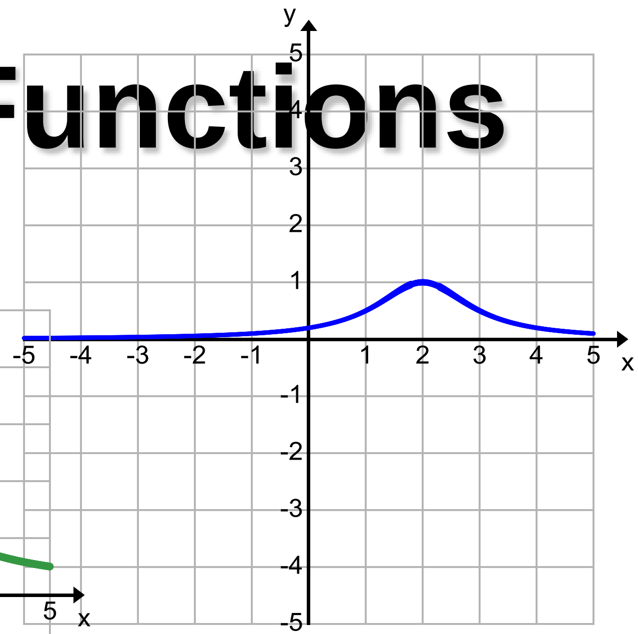
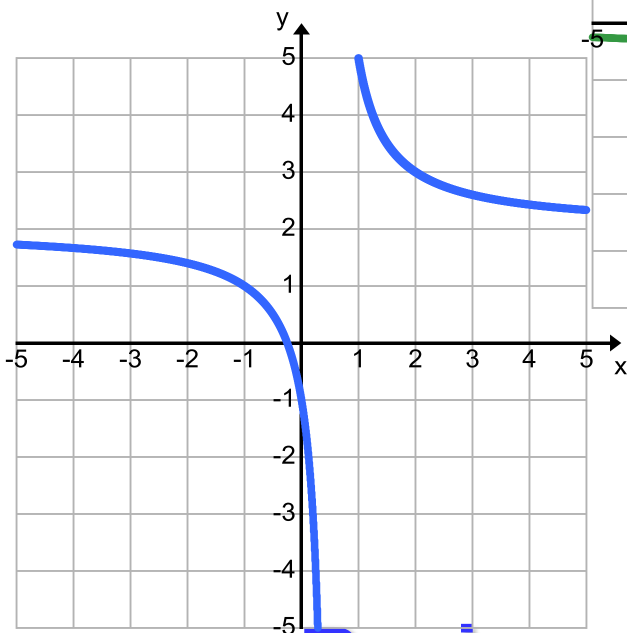
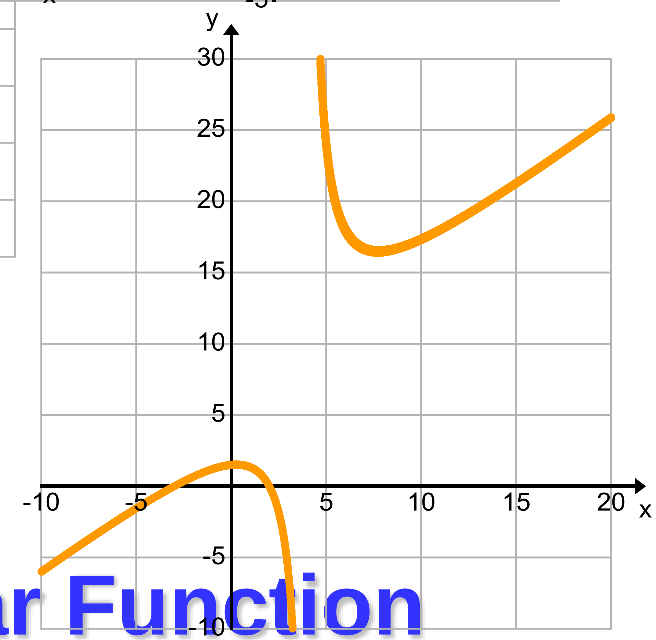
UNIT 2: Rational Functions











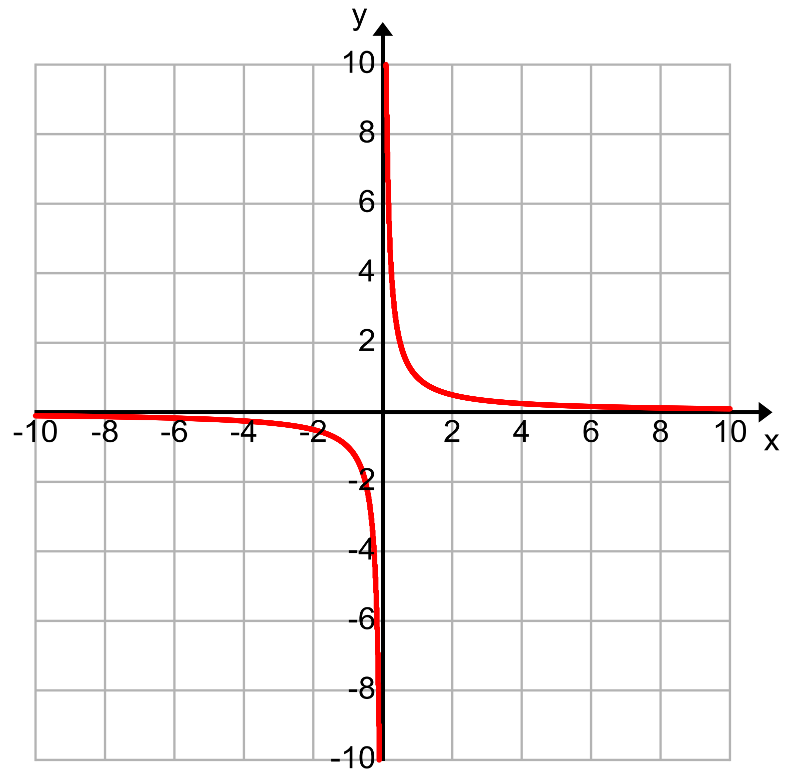
Reciprocal of a Linear Function

A Reciprocal Function

- Sketch the function $y = 1/x$.
- This is a Rational Function that is the reciprocal of $y = x$.
- Describe the basic features of this graph. Make reference to intercepts, asymptotes, end behaviour, local maximum and local minimums.

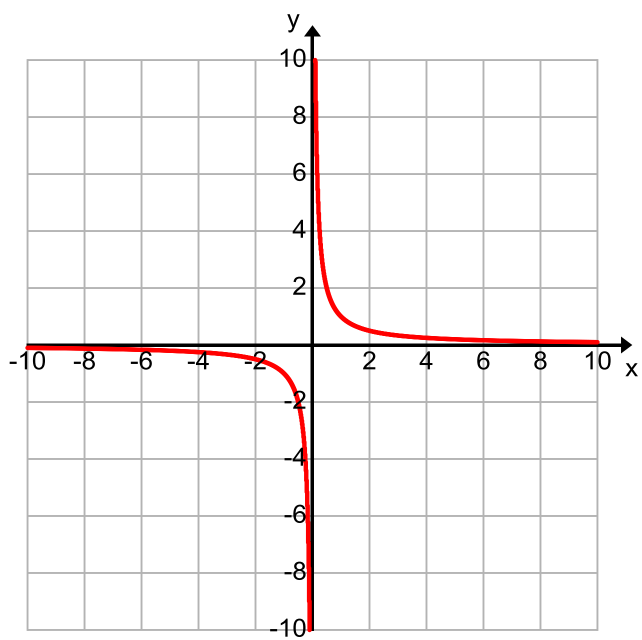
Properties of 1/x

- No intercepts
- Asymptotes at the x- and y-axis
- 1st and 3rd quadrant
- Decreasing Behaviour



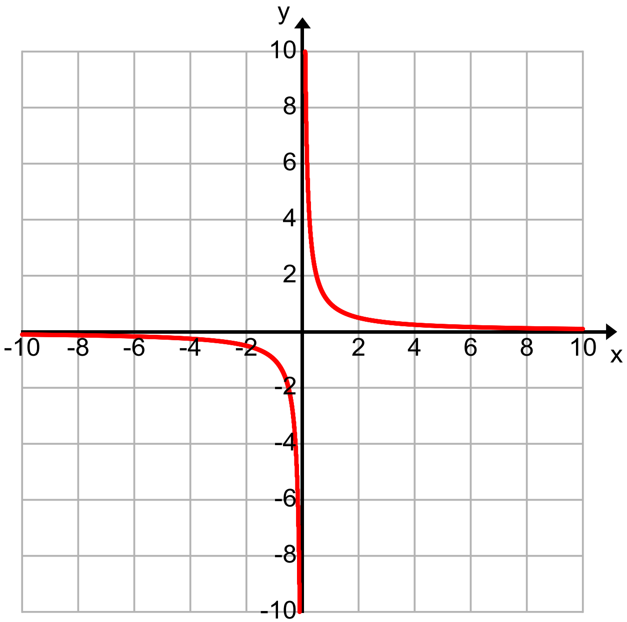
End Behaviour of a Rational Function

- What happens as x → ∞?
- What happens as x → -∞?



What happens as x approaches 0…

- From the left?
- From the right?



Describing the behaviour of a function as x approaches key values (See p. 153 #1)

x	$f(x)$
0^+	$+\infty$
0^-	$-\infty$
$+\infty$	0
$-\infty$	0

0^+ means x approaching zero from the right

0^- means …. From the left

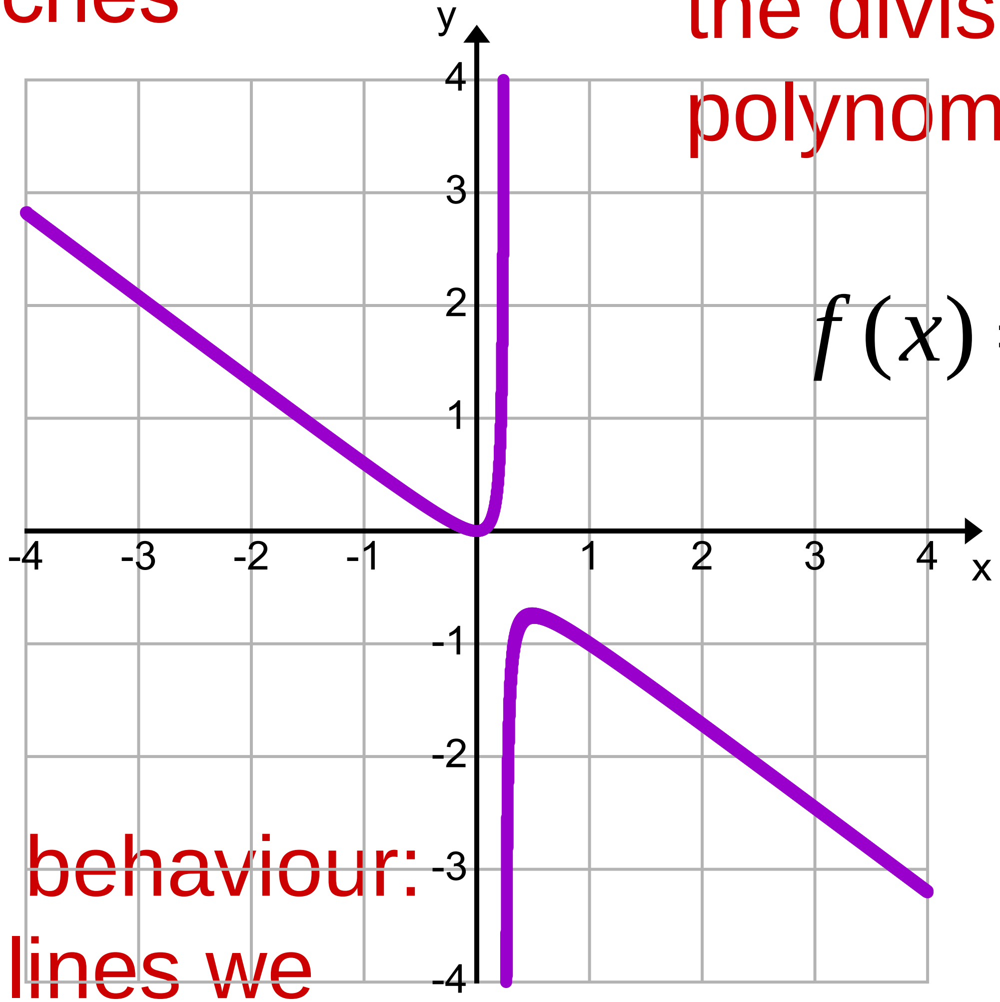
What is a rational function?

- How do you know it is not a polynomial function?

$$y = \frac{1}{3x - 1} \qquad y = \frac{3x^2}{1 - 4x} \qquad y = -\frac{x + 1}{2x^2 - 3}$$

There are 2 or more “branches”

The equation is the division of 2 polynomials



$$f(x) = \frac{3x^2}{1-4x}$$

Asymptotic behaviour:

- There are lines we just don’t cross…

What are the restrictions on rational functions?

- Example – Consider the rational function
  - Find the restrictions by setting the polynomial factors of the denominator to zero
  - These “restrictions” represent the **vertical asymptotes** of the graph

$$f(x) = \frac{3}{4-x}$$

$$4 - x = 0$$

$$x \neq 4$$

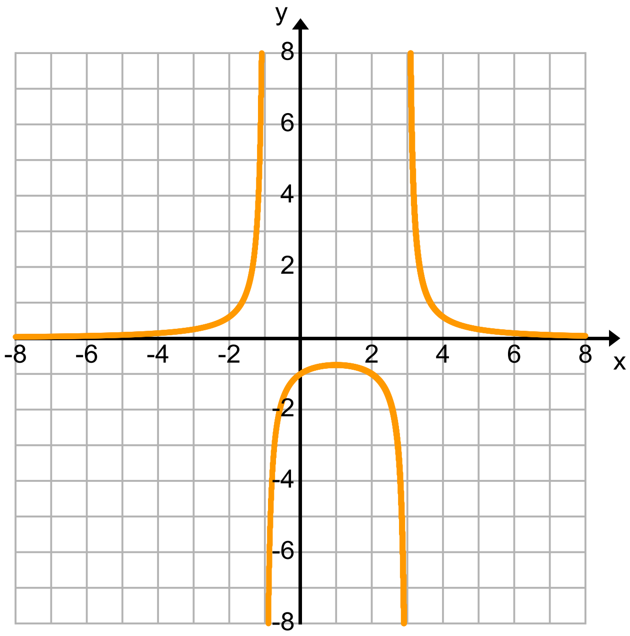
Try another one

$$f(x) = \frac{3}{x^2 - 2x - 3}$$

What are the restrictions on this function?

$$f(x) = \frac{3}{(x+1)(x-3)}$$

$$x \neq -1, 3$$



What do the restrictions tell you about the function?

Definition of a Rational Function

A **rational function** has the form h(x) = $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomials

The domain of a rational function consists of all real number except the zeroes of the polynomial in the denominator. $g(x) \neq 0$

The zeros of h(x) are the zeroes of f(x) if h(x) is in simplified form.

Finding the Domain and the Intercepts

- What is the domain of the rational function?
- Determine the x- and y-intercepts.
- Determine the domain and range.
- Describe the behaviour around key values.

$$f(x) = \frac{7}{x+2}$$

Finding the Domain and the Intercepts

x-intercept?

Set y to zero

$$0 = \frac{7}{x+2}$$

$$0(x+2) = 7$$

$$0 = 7$$

Not Possible!
Therefore, there is no x-intercept

y-intercept?

Set x to zero

$$y = \frac{7}{0+2}$$

$$y = 3.5$$

The y-intercept is at (0, 3.5)

Domain?

$$x \neq -2$$

$$x \in \Re$$

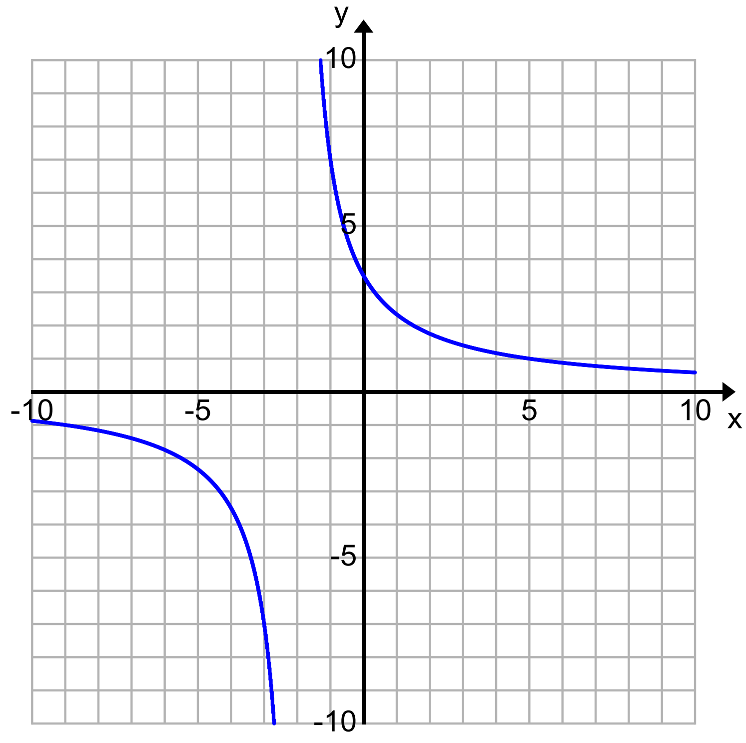
Range?

$$y \neq 0$$

$$y \in \Re$$

Consider the restrictions or asymptotes

$$f(x) = \frac{7}{x+2}$$



x	$f(x)$
-2^{+}	
-2^{-}	
$+\infty$	
$-\infty$	

A little about Asymptotes…

- **Vertical and Horizontal Asymptotes**
- The graph of a rational function has at least one asymptote, which may be vertical, horizontal, or oblique. An oblique asymptote is neither vertical nor horizontal.
- **Vertical Asymptote** can be found where the function is undefined. You'll find the vertical asymptotes algebraically by setting the denominator equal to zero and solving. The solutions will be the vertical asymptotes.
- Since the vertical asymptote is opposite to the domain. The solutions will be the values that are not allowed in the domain.
- **Horizontal Asymptote** indicates the general behaviour far off to both sides of the graph. It can be found by setting up a chart for large negative and positive values of x.
- **The graph of a rational function never crosses a vertical asymptote but it may or may not cross a horizontal asymptote.**

PROBLEMS

– AF p. 146 # 2
– AF p. 153 # 2, 3 ,5 , 9, 11,12